Proceedings of the
1999 ASME Design Engineering Technical Conferences
September 12-15, 1999, Las Vegas, Nevada

DETC99/DAC-8707

A SECOND-ORDER METHOD FOR ASSEMBLY TOLERANCE ANALYSIS

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ABSTRACT

Linear analysis and Monte Carlo simulation are two well-established methods for statistical tolerance analysis of mechanical assemblies. Both methods have advantages and disadvantages. The Linearized Method, a form of linear analysis, provides fast analysis, tolerance allocation, and the capability to solve closed loop constraints. However, the Linearized Method does not accurately approximate nonlinear geometric effects or allow for non-normally distributed input or output distributions. Monte Carlo simulation, on the other hand, does accurately model nonlinear effects and allow for non-normally distributed input and output distributions. Of course, Monte Carlo simulation can be computationally expensive and must be re-run when any input variable is modified.

The second-order tolerance analysis (SOTA) method attempts to combine the advantages of the Linearized Method with the advantages of Monte Carlo simulation. The SOTA method applies the Method of System Moments to implicit variables of a system of nonlinear equations. The SOTA method achieves the benefits of speed, tolerance allocation, closed-loop constraints, non-linear geometric effects and non-normal input and output distributions. The SOTA method offers significant benefits as a nonlinear analysis tool suitable for use in design iteration.

A comparison was performed between the Linearized Method, Monte Carlo simulation, and the SOTA method. The SOTA method provided a comparable nonlinear analysis to Monte Carlo simulation with 10^6 samples. The analysis time of the SOTA method was comparable to the Linearized Method.

1. INTRODUCTION

Tolerance analysis is increasingly becoming an important tool for mechanical design. This seemingly arbitrary task of assigning tolerances can have a large effect on the cost and performance of manufactured products. With the increase in competition in today's marketplace, small savings in cost or small increases in performance may determine the success of a product.

This paper proposes a new second-order tolerance analysis (SOTA) method. The development of the SOTA method was motivated by the differences in capabilities between two well-established tolerance analysis methods: the Linearized Method and Monte Carlo simulation. The SOTA method specifically addresses tolerance analysis of vector-loop tolerance models. The following three sections introduce vector-loop tolerance models, the Linearized Method and Monte Carlo simulation.

1.1 Vector-loop Tolerance Models

Vector loops can be used to model manufactured assemblies. Figure 1 shows an example of a two-dimensional assembly described by three vector loops. A vector-loop tolerance model mathematically establishes how the manufactured lengths and angles of each component combine in order to properly assemble together. The vector loops are able to model dimensional, form and kinematic variations.

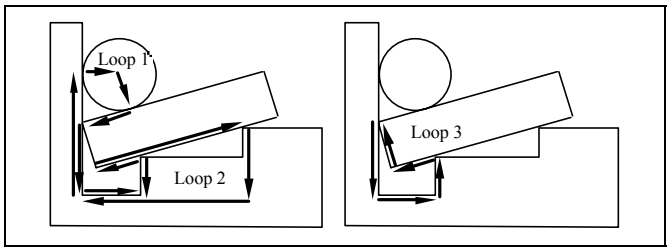

Figure 1: Vector-loop Assembly Model

Vector-loop closure is an important condition for assembly tolerance analysis. Closure simply refers to the condition when the beginning of the vector loop is the same position and orientation as the end of the loop. Loop closure is the mathematical equivalent of an assembly fitting together with no

clearance between parts. The loop closure condition can be written as the system of nonlinear equations:

$$\mathbf{h(x,u) = 0} \quad (1)$$

where **h** is the system of loop equations, **x** is the set of vectors representing manufactured component dimensions, and **u** is the set of vectors representing unknown assembly lengths and angles. The unknown assembly lengths and angles are the kinematic assembly dimensions that change as a function of the component dimensions.

1.2 Linearized Method

The *Linearized Method* is a vector-loop-based method of assembly tolerance analysis. The method's name comes from the fact that the nonlinear equations of the vector-loop model are linearized for the analysis. The linearized equations determine how small changes of the component dimensions, form and contact affect an assembly. For this method only one assembly needs to be analyzed statistically. Linear analysis is extremely fast and allows for tolerance allocation and design iteration. It is, however, limited to normal component distributions and cannot be applied to non-normal assembly distributions.

When tolerances are small compared to the nominal dimension, on the order of 1/100 to 1/1000, the Linearized Method gives excellent results. A comparison [Gao 1995] between the Linearized Method and Monte Carlo simulation found that the accuracy of the Linearized Method corresponded to Monte Carlo simulation with a sample size of 30,000, for quality levels near three sigma. However, for highly nonlinear assemblies or highly skewed distributions, the Linearized Method loses accuracy.

The Linearized Method expands the loop closure equation, Equation 1, for small variations about the nominal by Taylor's series expansion, retaining first order derivatives. This expansion yields:

$$dh_i = \sum_{j=1}^{n} \frac{\partial h_i}{\partial x_j} dx_j + \sum_{j=1}^{m} \frac{\partial h_i}{\partial u_j} du_j = 0 \quad (2)$$

where dx_j are the specified tolerances of the component dimensions and du_j are the resultant variations in the dependent assembly dimensions. This expression can be put in vector form by forming the matrix **A** of partial derivatives $\frac{\partial h_i}{\partial x_j}$ and the matrix **B** of the partial derivatives $\frac{\partial h_i}{\partial u_j}$.

$$[\ \mathbf{A}\ ]\ \{\mathbf{dx}\} + [\ \mathbf{B}\ ]\ \{\mathbf{du}\} = \{\ \mathbf{0}\ \} \quad (3)$$

Solving for **du**:

$$\{\mathbf{du}\} = -[\ \mathbf{B^{-1}}\ ]\ [\ \mathbf{A}\ ]\ \{\mathbf{dx}\} \quad (4)$$

Therefore, the product of the matrices $-\mathbf{B^{-1}A}$ gives the sensitivities of the dependent assembly dimension with respect to the component dimensions. Having established this relationship, the Standard Deviation of the dependent assembly dimension variations may be estimated by the root sum squares expression:

$$du_i = \sqrt{\sum_{j=1}^{n} \left(\frac{\partial u_i}{\partial x_j} dx_j \right)^2} \quad (5)$$

where $\frac{\partial u_i}{\partial x_j}$ are the elements of the $-\mathbf{B^{-1}A}$ matrix.

The formulation of the Linearized Method allows the implicit assembly dimensions in the loop equations to be expressed as an explicit, statistical function of the component dimensions.

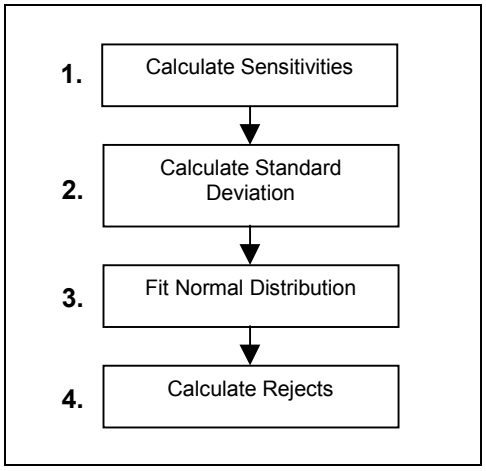

Figure 2: Steps of the Linearized Method

Figure 2 shows the steps of the Linearized Method. Step 1 is the calculation of the sensitivities, the elements of the $-\mathbf{B^{-1}A}$ matrix. Step 2 uses Equation 5 to calculate the Standard Deviation. Steps 3 applies a Normal distribution assumption to the Standard Deviation calculated in Step 2. Finally, Step 4 calculates the rejects given the Normal distribution and specification limits.

1.3 Monte Carlo Simulation

Monte Carlo simulation is a random number based method for performing assembly tolerance analysis. The manufacture of an assembly is simulated, for example, by creating a set of component dimensions with small random changes to simulate natural process variations. Next, the resulting assembly dimensions are calculated from the simulated set of component dimensions. The number of rejects that fall outside the specification limits are then counted. These three steps are illustrated in Figure 3.

Sample sizes generally range between 5,000 to 100,000 based on the required accuracy of the simulation. The accuracy

of Monte Carlo simulation increases with larger sample sizes. Obviously, the computational effort of large sample sizes can be significant, but Monte Carlo simulation offers many advantages because of its flexibility. Monte Carlo simulation allows any component distribution to be specified and will calculate the resulting assembly distribution.

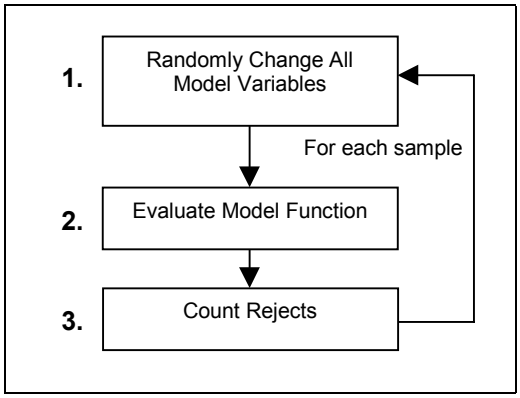

Figure 3: Steps of Monte Carlo Simulation

Monte Carlo simulation and the Linearized Method provide different capabilities. The Linearized Method can perform an analysis and a tolerance allocation quickly, so it is suitable for design iteration. The Linearized Method is limited in that it cannot output non-normal distributions or handle non-normal component distributions. Also, the Linearized Method will not be accurate for highly nonlinear assemblies. Monte Carlo simulation allows non-normal input distributions and a nonlinear analysis. However, Monte Carlo simulation is computationally expensive and does not accommodate rapid design iteration. For example, if a single input parameter is modified, the entire Monte Carlo simulation must be re-run.

Table 1: Comparison of Method Features

Features	Linearized Method	Monte Carlo Simulation	SOTA Method
Speed	√		√
Tolerance allocation	√		√
Closed-loop constraints	√		√
Nonlinear approximation		√	√
Non-normal input distributions		√	√
Non-normal output distributions		√	√

Table 1 summarizes the features of the Linearized Method, Monte Carlo simulation and the Second-Order Tolerance Analysis (SOTA) method proposed in this paper. The SOTA method attempts to combine the features of the Linearized Method and Monte Carlo simulation.

The next section of this paper, Section 2, discusses research related to the SOTA method. Section 3 presents the SOTA method. Section 4 compares the results of the SOTA method with the Linearized Method and Monte Carlo simulation for a sample problem.

2. RESEARCH REVIEW

2.1 Linearized Method

The Linearized Method, explained in Section 1.2, provides a quick way to perform nonlinear tolerance analysis for both explicit and implicit assembly dimensions of a vector-loop tolerance model. Because of its speed, the Linearized Method is ideal for design iteration and tolerance allocation. Multiple research studies have continued to refine the Linearized Method, making it more general and accurate.

The Direct Linearization Method (DLM) [Marler 1988] prescribed a systematic approach to vector-loop model tolerance analysis. DLM has enabled the Linearized Method to be applied to a broad range of tolerance problems. Most importantly, DLM has allowed a general tolerance analysis methodology to be incorporated into a computer program suitable for integration with a CAD system.

More recently, the Global Coordinate Method [Gao 1993] for determining the partial derivatives of the loop equations was developed. This method simplified the calculations of these derivatives. In the same paper, Gao benchmarked the Linearized Method against a comparable Monte Carlo simulation system. The benchmark results showed that the accuracy of the Linearized Method corresponds to Monte Carlo simulation with a sample size of 30,000 for quality levels of three sigma.

The Linearized Method has demonstrated its usefulness as a design tool. However, the method is inadequate for highly nonlinear tolerance problems and non-normal input distributions.

2.2 Monte Carlo Simulation

Generally, Monte Carlo simulation is applied to an *explicit* function of random variables. However, the variables of interest in the equations of a vector-loop tolerance model are inherently *implicit*. McCATS, a Monte Carlo based tolerance analysis method developed recently [Gao 1995], is able to adequately handle the implicit equations of a vector-loop tolerance model.

The McCATS system starts by generating random variates for the assembly variables. These random variates are sent to an assembly function that then solves the nonlinear system of loop equations iteratively for the dependent assembly dimensions. The assembly dimensions are stored, new random variates are generated, and the assembly function is called again. This procedure is continued until the desired number of assemblies has been simulated. Solving the loop equations iteratively for each assembly simulation is critical to the accuracy of the tolerance model.

Including the capability for kinematic constraints in Monte Carlo simulation enabled Monte Carlo methods to be applied to a much broader range of design problems. However, the required iterative assembly function does add more calculations to an already computationally intense method.

2.3 Method of System Moments

The Method of System Moments (MSM) [Cox 1979, Shapiro 1981] is a technique for estimating system output based upon the relationship between input and output variables and information about the distribution of the inputs. MSM is also known as nonlinear propagation of error and propagation of moments. MSM estimates the first four moments of a function of random variables. The first four statistical moments are show in Figure 4.

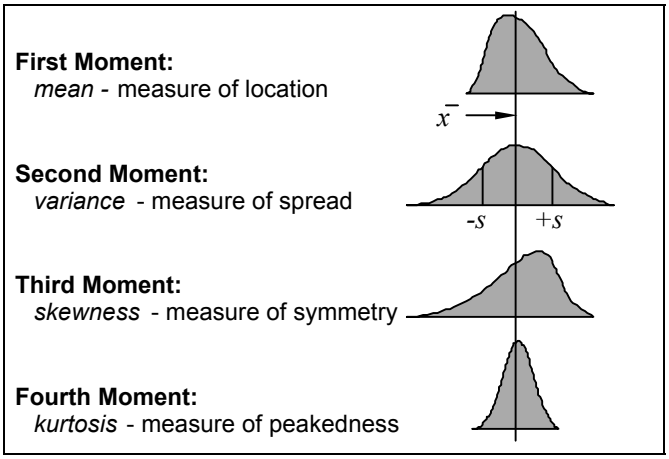

Figure 4: First Four Statistical Moments

MSM is formulated by expanding the function of interest in Taylor series about its mean values. Retaining second-order terms the expansion yields:

$$u_i = \bar{u}_i + \sum_{j=1}^{n} \frac{\partial u_i}{\partial x_j}\left(x_j - \bar{x}_j\right) + \frac{1}{2}\sum_{j=1}^{n} \frac{\partial^2 u_i}{\partial x_j^2}\left(x_j - \bar{x}_j\right)^2 + \sum_{j=1}^{n-1}\sum_{k=j+1}^{n} \frac{\partial^2 u_i}{\partial x_j \partial x_k}\left(x_j - \bar{x}_j\right)\left(x_k - \bar{x}_k\right) \tag{6}$$

The approximate system mean, the first distribution moment, is calculated by taking the expected value of the above expression, which gives:

$$E(u_i) = \bar{u}_i + \frac{1}{2}\sum_{j=1}^{n} \frac{\partial^2 u_i}{\partial x_j^2}\operatorname{var}(x_j) \tag{7}$$

The number of terms in the expressions for the higher moments increases dramatically. For instance, the second, third and fourth moments require that the expected value be found for Equation 6 raised to the second, third and fourth power. The expressions for the higher moments are simplified if the origin is shifted to the mean values. In terms of the new notation, the first four moments of the assembly dimensions are approximated by the following four equations:

$$m_1(u_i) = \sum_{j=1}^{n} b_{jj}\mu_2(x_j) \tag{8}$$

$$m_2(u_i) = \sum_{j=1}^{n}\left[b_j^2\mu_2(x_j) + 2b_j b_{jj}\mu_3(x_j) + b_{jj}^2\mu_4(x_j)\right] + \sum_{j=1}^{n-1}\sum_{k=j+1}^{n}\left[2b_{jj}b_{kk} + b_{jk}^2\right]\mu_2(x_j)\mu_2(x_k) \tag{9}$$

$$m_3(u_i) = \sum_{j=1}^{n} b_j^3\mu_3(x_j) \tag{10}$$

$$m_4(u_i) = \sum_{j=1}^{n} b_j^4\mu_4(x_j) + \sum_{j=1}^{n-1}\sum_{k=j+1}^{n} 6b_j^2 b_k^2\mu_2(x_j)\mu_2(x_k) \tag{11}$$

Where,

$$b_j = \frac{\partial u_i}{\partial x_j} \quad b_{jj} = \frac{1}{2}\frac{\partial^2 u_i}{\partial x_j^2} \quad b_{jk} = \frac{\partial^2 u_i}{\partial x_j \partial x_k}$$

The term $\mu_i(x_j)$ represents the *ith* distribution moment of the *jth* component dimension.

Equations 10 and 11 have been truncated significantly in order to simplify the expressions. The complete third moment equation is lengthy, and the complete fourth moment equation is formidable. The complete equations for the third and fourth moments may be found in [Cox 1979].

After calculating Equations 8 through 11, the four moments about the mean may be found from:

$$E(u_i) = m_1(u_i) + \bar{u}_i \tag{12}$$

$$E(u_i^2) = m_2(u_i) - [m_1(u_i)]^2 \tag{13}$$

$$E(u_i^3) = m_3(u_i) - 3m_2(u_i)m_1(u_i) + 2[m_1(u_i)]^3 \tag{14}$$

$$E(u_i^4) = m_4(u_i) - 4m_3(u_i)m_1(u_i) + 6m_2(u_i)[m_1(u_i)]^2 - 3[m_1(u_i)]^4 \tag{15}$$

To estimate the four moments of an assembly distribution using the full quadratic model requires the first eight moments of the component dimension distributions and the partial derivatives $\frac{\partial u_i}{\partial x_j}$, $\frac{\partial^2 u_i}{\partial x_j^2}$ and $\frac{\partial^2 u_i}{\partial x_j \partial x_k}$.

In a comparison of advanced tolerance analysis methods [Greenwood 1987], the Method of System Moments was recommended as the best method.

3. THE SOTA METHOD

The second-order tolerance analysis (SOTA) method is proposed as a general analysis method for vector-loop tolerance models. The SOTA method is comprised of a nonlinear system solver, finite difference approximations for the first and second order partial derivatives, the Method of System Moments (MSM), and a Generalized Lambda Distribution (GLD) empirical fit. The difference equations and nonlinear solver are used together to supply MSM with the required relationships between the component dimensions and the resultant assembly dimensions. MSM is then used to calculate the first four moments of the assembly dimensions. Finally, GLD is used to fit the calculated moments and approximate the distribution of

the assembly dimensions. The SOTA method process is shown in Figure 5.

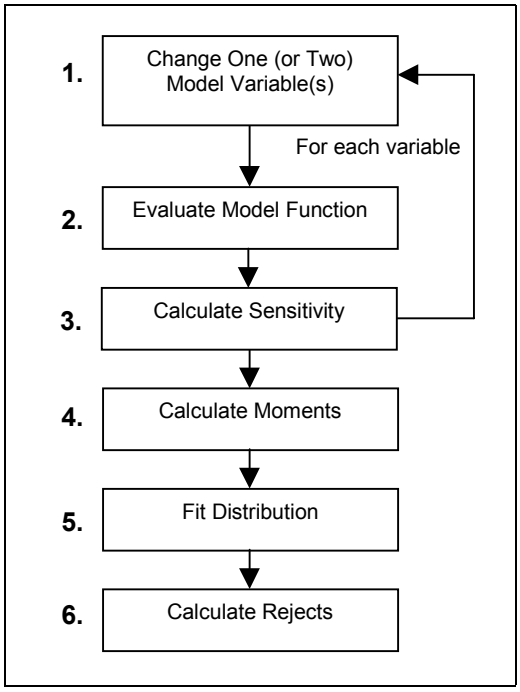

Figure 5: Steps of the SOTA Method

3.1 Difference Formulas

In order to approximate the three sets of partial derivatives, $\frac{\partial u_i}{\partial x_j}$, $\frac{\partial^2 u_i}{\partial x_j^2}$ and $\frac{\partial^2 u_i}{\partial x_j \partial x_k}$, three separate difference formulas are required. The linear partial derivatives are approximated by a central difference formula:

$$\frac{\partial u_i}{\partial x_j} \approx \frac{u_i(x_j + \Delta x_j, u_i) - u_i(x_j - \Delta x_j, u_i)}{2\Delta x_j} \qquad (16)$$

For this notation, $u_i(x_j + \Delta x_j, u_i)$ represents a function evaluation for the implicit assembly dimensions u_i, where the component dimensions, x_j, are at their nominal value except for the *jth* dimension, which is perturbed by a value Δx_j. So, Equation 16 indicates two function evaluations for each component dimension x_j. Note that each function evaluation requires an iterative solution of a system of nonlinear equations.

A three-point difference formula is used for the approximation of the quadratic partial derivatives [Burden 1993].

$$\frac{\partial^2 u_i}{\partial x_j^2} \approx \frac{u_i(x_j + \Delta x_j, u_i) - 2u_i(x_j, u_i) + u_i(x_j - \Delta x_j, u_i)}{\Delta x_j^2} \qquad (17)$$

Two of the function evaluations that appear in this three point difference formula also appear in the central difference formula, Equation 16. Therefore, if there are *n* component dimensions, *2n* function evaluations can be avoided if the same function evaluations are used for both difference Equations 16 and 17. If this is done, the quadratic partial derivatives will require *2n* function evaluations plus one evaluation at the nominal and, without any further evaluations, the linear partials can also be obtained.

The approximation of the cross-derivatives is more complicated. These partial derivatives are found by using the central difference of a central difference.

$$A = \frac{u_i(x_j + \Delta x_j, x_k + \Delta x_k, u_i) - u_i(x_j - \Delta x_j, x_k + \Delta x_k, u_i)}{2\Delta x_j}$$

$$B = \frac{u_i(x_j + \Delta x_j, x_k - \Delta x_k, u_i) - u_i(x_j - \Delta x_j, x_k - \Delta x_k, u_i)}{2\Delta x_j}$$

$$\frac{\partial^2 u_i}{\partial x_j \partial x_k} \approx \frac{(A - B)}{2\Delta x_k} \qquad (18)$$

For *n* component dimensions there are $(n^2-n)/2$ unique cross-derivatives. Four new function evaluations must be performed for each derivative. Therefore, $2n^2-2n$ evaluations are required to obtain the cross-derivatives. Together with the $2n+1$ function evaluations for the linear and quadratic partial derivatives, the total becomes $2n^2+1$ function evaluations. Thus, with $2n^2+1$ function evaluations, the required partial derivatives for the SOTA method are obtained.

3.2 Distribution Fit

The Generalized Lambda Distribution was chosen as the best empirical model for fitting the statistical moments based on ease of implementation. The single form of the GLD make the method of matching of moments easily applied to the moments calculated by MSM, whereas, the Johnson and Pearson systems require multiple distribution forms to cover a full range of moments. In addition, a GLD table, indexed by skewness and kurtosis values, was readily available for use in a computer program because of earlier research [Gao 1995]. The GLD's range of coverage is smaller than the Johnson and Pearson systems, however, it does cover most practical distribution shapes likely to be encountered in mechanical assemblies.

4. EXAMPLE

The following One-Way Clutch example problem illustrates the performance of the SOTA method compared to Monte Carlo Simulation and the Linearized Method. The example problem was analyzed using the SOTA method, the Linearized Method and Monte Carlo simulation at four different sample sizes. The four sample sizes were 30,000 samples, 100,000 samples, 10^6 samples and 10^9 samples.

A one-way clutch transmits torque in a single direction. The clutch assembly consists of the following components: a hub, an outer ring, four rollers, and four springs. When the hub rotates in a counter-clockwise direction, the roller wedges between the hub and the ring, locking these two parts together. When the hub turns in a clockwise direction, the spring is compressed by the roller, the roller slips, and the hub is allowed to rotate freely. The one-way clutch assembly and the single vector loop used to model this assembly are shown in Figure 6.

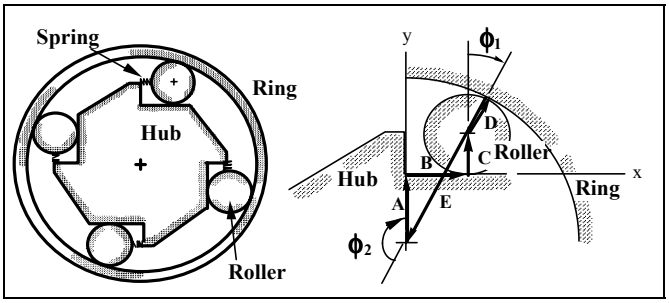

Figure 6: One-Way Clutch Assembly

The function of the one-way clutch mechanism is governed by the pressure angle ϕ_1. There are three manufactured dimensions that control the pressure angle. The mean values, standard deviations and distribution types of these three dimensions are shown in Table 2. The vectors representing the roller radius, vectors **C** and **D**, were treated as the same variation source. As the pressure angle, ϕ_1, is critical to the function of the clutch, it was given specification limits as shown in Table 3.

Table 2: Input Variables

Name	Mean	Standard Deviation	Distribution
A	27.645 mm	0.01666 mm	Normal
C, D	11.430 mm	0.00333 mm	Normal
E	50.800 mm	0.00416 mm	Normal

Table 3: Assembly Specification

Name	Nominal	Lower Limit	Upper Limit
ϕ_1	7.0184°	6.4184°	7.6184°

4.1 Analysis Results

The One-Way Clutch analysis results are displayed in Table 4 and Table 5. Table 4 shows the calculated values for the first four statistical moments of the pressure angle. The One-Way Clutch assembly was a good test problem since the pressure angle exhibits nonlinear behavior. The results show the pressure angle to be negatively skewed and slightly more peaked than a Normal distribution. Because all the input distributions were symmetric, this skewness indicates that the pressure angle is an inherently nonlinear function. Of course, the skewness value calculated by the linear analysis was zero since the linear analysis cannot estimate this non-linearity.

Table 5 contains the predicted parts-per-million (PPM) assemblies that fall outside of the specification limits of the pressure angle. All the Total Rejects results were within 1000 PPM of the Monte Carlo simulation of 10^9 samples.

Table 4: Statistical Moments Results

Analysis	Mean	Standard Deviation	Skewness	Kurtosis
MC 1e9	7.014953	0.219668	-0.09442	3.023816
MC 1e6	7.015373	0.219884	-0.09477	3.027695
MC 100k	7.015453	0.220172	-0.10168	3.021511
MC 30k	7.012982	0.220541	-0.09758	3.082748
SOTA	7.014968	0.219346	-0.09356	3.011671
Linear	7.018389	0.219292	0	3

Table 5: PPM Rejects Results

Analysis	Lower Rejects	Upper Rejects	Total Rejects
MC 1e9	4406	2166	6572
MC 1e6	4467	2206	6673
MC 100k	4580	2080	6660
MC 30k	5000	2567	7567
SOTA	4196	2322	6518
Linear	3109	3109	6218

In order to compare the results of the six analyses, a relative error measure was calculated for the estimates of the statistical moments and the estimate of PPM rejects. Monte Carlo Simulation with 10^9 samples was assumed to be the most accurate analysis and was, therefore, used as the baseline for the relative error comparison.

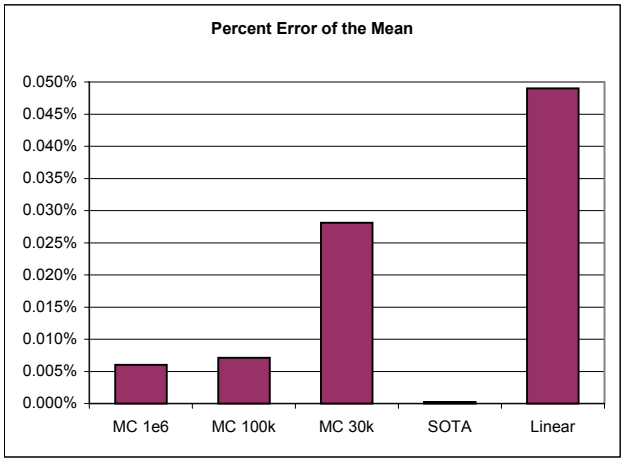

Figure 6: Error of the Mean

Figure 6 compares the error of the pressure angle mean with respect to the Monte Carlo 10^9 analysis. All five analyses estimated the mean very accurately to within 0.05% error. The SOTA method was the most accurate with only 0.0002% error.

Figure 7 displays the error of the standard deviation. The standard deviation values were also very accurate with the error ranging from 0.10% for the Monte Carlo 10^6 to 0.40% for the Monte Carlo 30k. Both Figure 6 and Figure 7 clearly illustrate how Monte Carlo Simulation should increase in accuracy as the sample size increases.

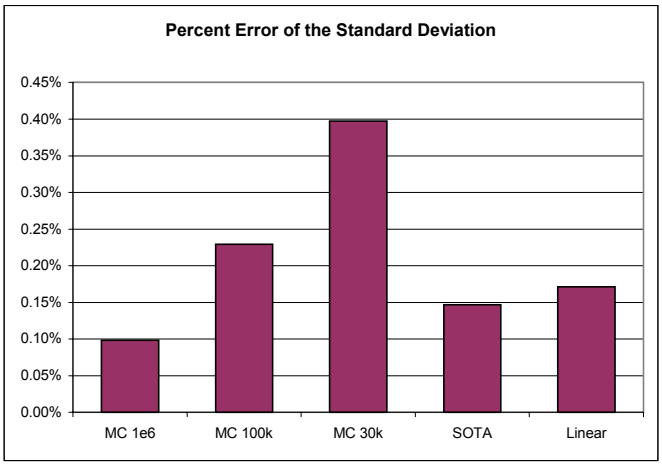

Figure 7: Error of the Standard Deviation

The truncation of nonlinear terms of the linear analysis is evident in the skewness results. With symmetric input distributions, a linear analysis will always predict a skewness value of zero. Figure 8 shows the absolute error of the skewness. With the exception of the linear result all skewness values are relatively accurate.

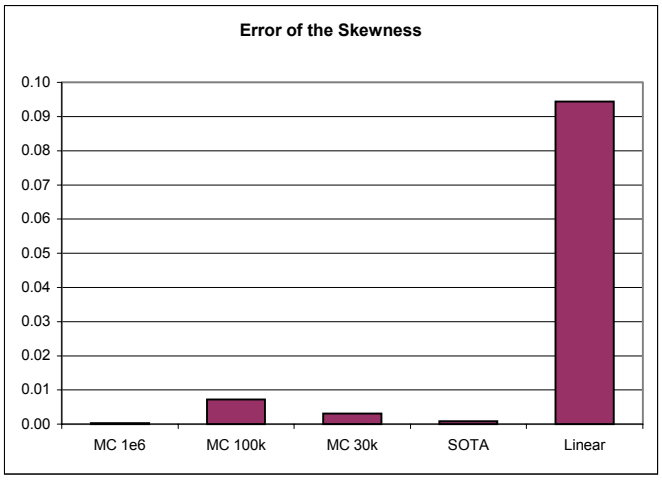

Figure 8: Error of the Skewness

The percent error of the kurtosis values is shown in Figure 9. With the exception of the Monte Carlo 30k result, all the kurtosis values had errors under 1%.

The process of calculating rejects for an assembly specification involves the four statistical moments and fitting a distribution to these moments. The Generalized Lambda Distribution was fit to the four moments in all six analyses. The reject results are a composite result of all four statistical moment estimates and, therefore, provide a good overall measure of accuracy for an analysis.

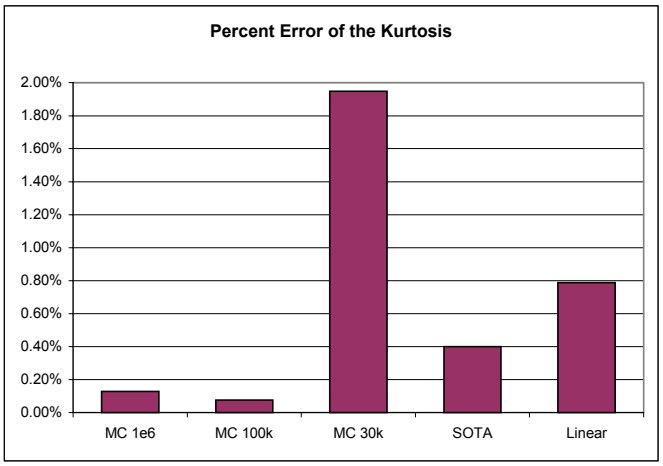

Figure 9: Error of the Kurtosis

Figure 10 shows the error of the upper, lower and total PPM rejects relative to the Monte Carlo 10^9 analysis. The linear analysis predicted symmetric rejects: 3109 ppm for the lower limit and 3109 ppm for the upper limit. The linear appoximation of the nonlinear pressure angle function resulted in an underestimate for the lower rejects and an overestimate for the upper rejects.

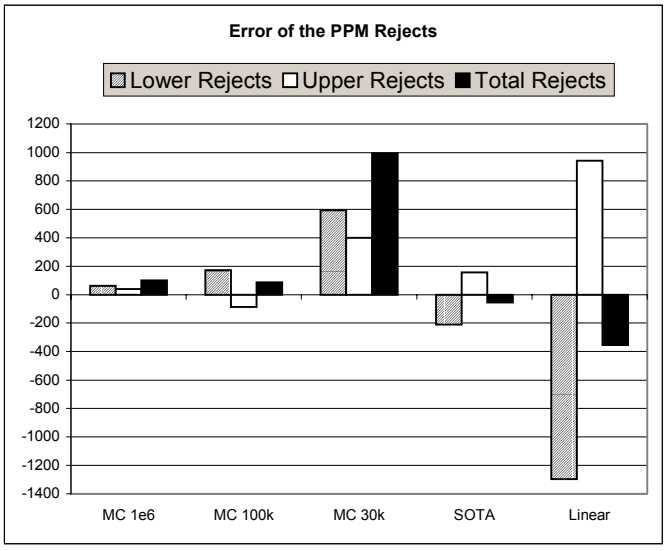

Figure 10: Error of the Rejects

The second-order approximation of the SOTA method dramatically improved the estimate of rejects over the linear approximation. While the SOTA method still slightly underestimated the lower rejects and slightly overestimated the upper rejects, the total rejects estimate was within 54 ppm of

the Monte Carlo 10^9 analysis, roughly equivalent to the Monte Carlo 100k results.

4.2 Computational Effort

For the three methods, Monte Carlo simulation, the SOTA method and the Linearized Method, a relative measure of effort is easily formulated. The common operation of these three tolerance analysis methods is that each must perform a linear solution of the loop equations. For example, the SOTA method and Monte Carlo simulation require a linear solution of the loop equations for each iteration of Newton's method. Of course the Linearized Method only requires a single linear solution. So, if the Linearized Method is given an effort value of 1, the relative effort of Monte Carlo simulation and the SOTA method may be evaluated by the following expressions:

MC Effort = (sample size) x (average Newton iterations)
SOTA Effort = $(2n^2 + 1)$ x (average Newton iterations)

For the SOTA Effort expression the variable *n* is the number of component dimensions.

It would be expected that the number of iterations of Newton's method be greater for Monte Carlo simulation than for the SOTA method. For each nonlinear solution, Monte Carlo simulation changes the nominal value of all the component dimensions, whereas the SOTA method only changes one or two component dimensions for each solution. Furthermore, the step size used by the SOTA method will generally be very small compared to the variations required by Monte Carlo. The average number of Newton iterations along with the effort metrics for the six analyses is shown in Table 6.

Table 6: Relative Computational Effort

Analysis	Average Iterations	Effort
MC 1e9	3.40	3,400,000,000
MC 1e6	3.40	3,400,000
MC 100k	3.40	340,000
MC 30k	3.41	102,300
SOTA	2.16	41
Linear	1	1

5. CONCLUSIONS

The SOTA method is a general, nonlinear tolerance analysis method for vector loop tolerance models. The SOTA method provides the benefits of speed, tolerance allocation, closed-loop constraints, a nonlinear approximation and the capability for non-normal input and output distributions.

For the One-Way Clutch example problem the SOTA method shows a dramatic improvement in accuracy over the linear approximation for the estimates of the four statistical moments of the pressure angle. The estimate of total rejects result for the SOTA method was comparable to the Monte Carlo result using 10^6 samples. This accuracy level is significant since the computational effort of the SOTA method was five orders of magnitude less than the Monte Carlo simulation with 10^6 samples.

Seven additional example problems were analyzed [Glancy 1994] and demonstrated similar results.

REFERENCES

[Burden 1993] Burden, Richard L. and J. Douglas Faires, *Numerical Analysis Fifth Edition*, PWS Publishing Co., 1993

[Chase 1995] Chase, Kenneth W., J. Gao and S. P. Magelby. "General 2-D Tolerance Analysis of Mechanical Assemblies with Small Kinematic Adjustments*." J. of Design and Manufacturing*, 5 (1995): 263—274

[Cox 1979] Cox, N. D., "Tolerance Analysis by Computer", *J. of Quality Technology*, Vol. 11, No. 2, April 1979

[Cvetko 1997] Cvetko, Robert. "Characterization of Assembly Variation Analysis Methods", MS Thesis, Brigham Young University, Dec. 1997

[Dudewicz 1974] Dudewicz, E. J., J. S. Ramberg and P. R. Tadikamalla, "A Distribution for Data Fitting and Simulation", *An. Tech. Conf. Trans. A.S.Q.C.*, 28 (1974): 407—418

[Gao 1995] Gao, J., K. W. Chase and S. P. Magleby, "Comparison of Assembly Tolerance Analysis by the Direct Linearization Method and Modified Monte Carlo Simulation Methods", Proc. of the ASME Design Engineering Tech. Conf., 1995, 353—360

[Glancy 1994] Glancy, Charles G., "A Second-Order Method for Assembly Tolerance Analysis", MS Thesis, Brigham Young University, Dec. 1994

[Greenwood 1987] Greenwood, W. H., "A New Tolerance Analysis Method for Designers and Manufacturers", Dissertation, Brigham Young University, 1987

[Marler 1988] Marler, Jaren D., "Nonlinear Tolerance Analysis Using the Direct Linearization Method", MS Thesis, Brigham Young University, 1988

[Shapiro 1981] Shapiro, Samuel S., Alan J. Gross*, Statistical Modeling Techniques*, Marcel Dekker, Inc., New York, 1981

[Ramberg 1979] Ramberg, J. S., P. R. Tadikamalla, E. J. Dudewicz and E. F. Mykytha, "A probability distribution and its uses in fitting data", *Technometrics*, 21 (1979): 201—214