



## Incorporating Thermal Expansion into CAD-Based 3-Dimensional Assembly Variation Analysis

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## Abstract

Proper integration of third party, add-on software solutions brings great power and speed to product development. The software architecture behind CETOL 6 $\sigma$  Tolerance Analysis Software provides an efficient process to analyze assembly behavior at any operating temperature including assemblies with diverse material properties. This paper describes the integration of CETOL 6 $\sigma$  technology with CAD technology, how to create an accurate assembly variation model, and how to introduce thermal expansion into the CAD/CETOL 6 $\sigma$  system in order to analyze the fit and function of assemblies operating at any feasible operating temperature.

## Background

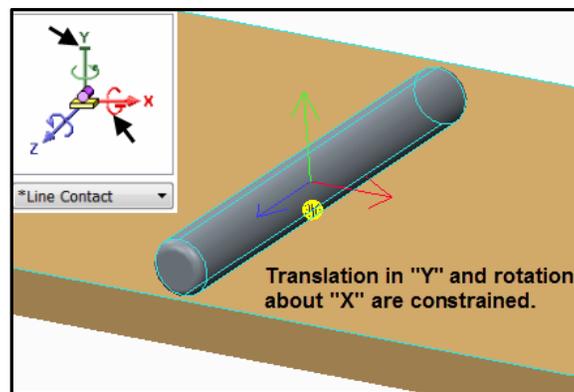
For most companies, assembly variation analysis is done utilizing simplifying assumptions in order to easily create calculations in spreadsheets. Analyzing complex assemblies using a 1-dimensional, “stack-up” approach requires the user to ignore 2D and 3D effects that would otherwise introduce leverage into the design. This leverage, also known as sensitivity, can significantly alter the location of parts with respect to each other resulting in reduced functionality and impacting the ability to assemble the product.

Instead of ignoring the 2D & 3D behaviors, spreadsheets can be programmed to project the 2D effects into the 1D direction using trigonometry but this adds to the complexity of the spreadsheet and is often avoided. Further, any changes to the CAD geometry can cause the formulas for 2D and 3D stacks to be out-of-sync and therefore erroneous. Avoidance can be due to a number of reasons including, 1) the mathematical nature of the problem 2) lack of standardized programs to efficiently complete the tolerance stacks, 3) time and confidence required to program a spreadsheet, and 4) schedule pressure which ultimately forces the designer to use the simplified approach in order to complete the task.

Solutions to the challenge of creating more complex analyses have been around for well over 20 years. With the introduction of 3D solid modeling packages like PTC Creo, SOLIDWORKS, CATIA, etc., the software development team behind CETOL 6 $\sigma$  has been able to take advantage of 3-dimensional surface information to create properly constrained assemblies based on the

kinematic constraint definition derived from functional assembly interfaces. Kinematic constraint definition refers to

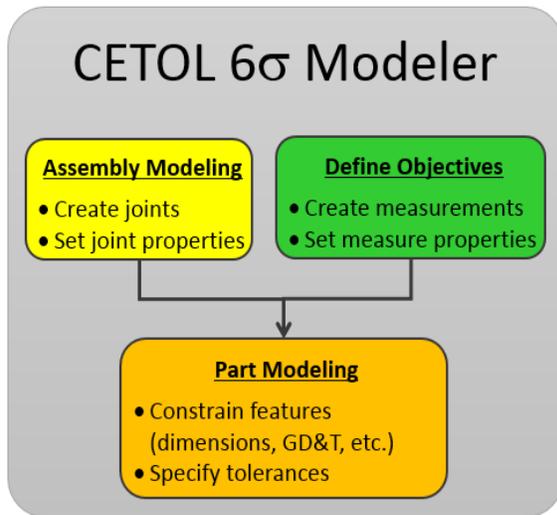
the natural degree of freedom condition that results when two features, “feature pairs”, are placed in contact. For example, a cylindrical surface (feature) on one part contacting a planar surface on another part would result in line contact. This contact by default constrains two degrees of freedom (DOFs) between the two parts, one translational DOF and one rotational DOF. See image below.



Coupling part contact behavior with the CAD geometry, a mathematical model can be constructed by the software to understand how surface variation will affect other key part relationships in the assembly. These key relationships, represented by “measurements” in CETOL  $6\sigma$ , are the “fit and function” requirements for the assembly and are usually derived out of some form of failure mode analysis, i.e. DFMEA, Design Failure Mode and Effect Analysis. The variation of surfaces is controlled by dimension schemes with assigned tolerances. For statistical analysis, the expected variation from the manufacturing process for the surface is factored in based on a  $C_p$  or  $C_{pk}$  value. Expected variation is then replaced with measured variation as the design approaches product release and inspection process validate supplier capabilities. Following is an example that goes into more detail on feature pairs, kinematic constraints, DOFs, and measurements.

## How to Create an Accurate Assembly Variation Model

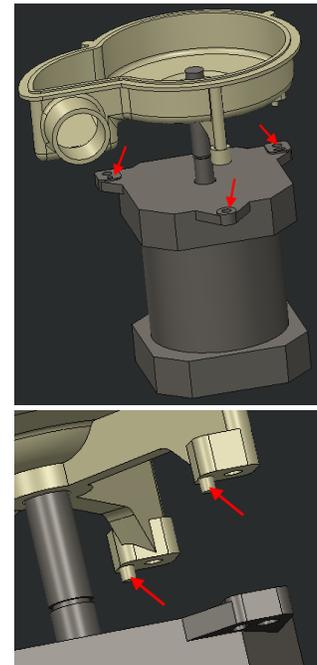
Studying assembly variation involves three basic data sources including: 1) functional assembly interfaces, 2) functional assembly requirements, and 3) part dimension schemes.



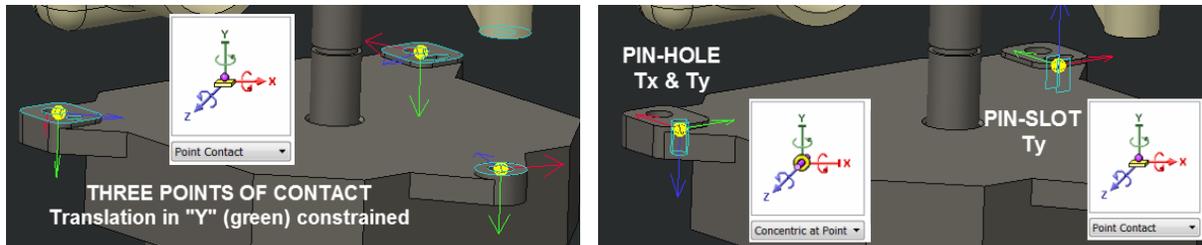
The diagram on the left shows the CETOL 6 $\sigma$  modeling process but this can be generalized into simple assembly engineering thinking. First, define precisely, using actual CAD surfaces, the true physical assembly conditions. CAD assembly models are typically an idealized representation. CETOL 6 $\sigma$  enables a precise representation of the actual assembly constraints. Secondly, what did the DFMEA say about what this product is supposed to do? Create relationships between parts that represents the requirements of the DFMEA. Finally, define the relationships between features within a part with either standard dimension or geometric tolerances, GD&T.

Take for example the pump-motor assembly shown on the right. The assembly modeling required must represent the physical conditions. The pump housing sits on top of three mounting features and locates using pins. The pins are inserted into one tight fitting hole and a tight fitting slot with the length of the slot "pointing" towards the hole. The first assembly interface is the contact with the three mounting pads on the motor. The features used are the small planar surfaces at the three locations. While this could be modeled with a single planar feature, the reality is that manufacturing variation, especially in the pump housing, assumed to be molded, results in different degrees of variation at each of the pump housing "legs". Therefore, the proper analysis setup would be to include all three legs independently with their own variation. This can be done using three points of contact.

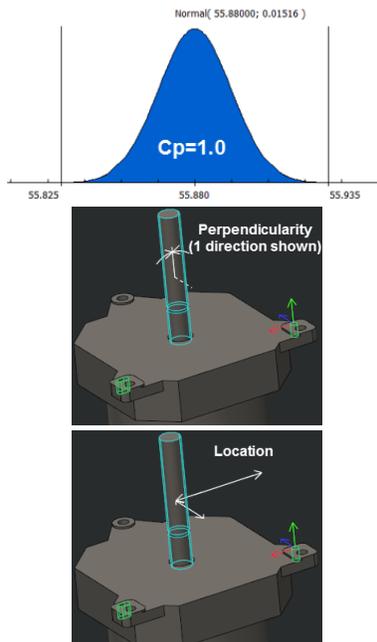
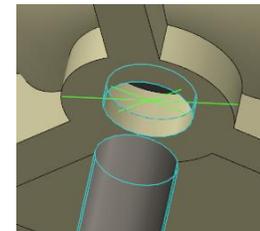
The next most controlling interface is the pin in the tight-fitting hole. The feature pair in this case is an external cylinder inside an internal cylinder. These features constrain the two in-plane translations, 2 DOFs, effectively leaving the pump housing free to spin about the axis of this pin. That's where the slot comes in. The third interface between the pin and the slot controls the clocking of the pump housing with respect to the motor.



The resulting kinematic degrees of freedom at these (5) interfaces would be: (3) point contact, one “concentric at point” contact controlling the two in-plane translations, and one point contact at the slot in the direction of the minor width. By defining the physical behavior using degrees of freedom, the location and orientation of the pump housing with respect to the motor will update based on the dimension schemes applied at the mounting pads, the two pins, the hole, and the slot. For example, as the location of the slot varies with respect to the hole in the motor, the pump housing must move to accommodate the variation. For completeness, the clearance between the pin-hole and pin-slot interfaces must also be considered to be a floating condition or biased where the pins are tangent in the hole and slot due to some external force.



The objective for this example analysis is to prevent leakage between the shaft and the pump housing. If the shaft decenters by more than 0.25 mm, the possibility of leakage exists. A measurement is created between the shaft and the center hole in the pump housing. Part and assembly process variation will affect the centering of the shaft.



Variation is introduced by the tolerance assigned to the feature-to-feature relationships within a part otherwise known as the dimension scheme. For example, the shaft on the motor could be located relative to the sides of the motor housing with simple linear dimensions and a tolerance. Alternatively, a positional geometric constraint could be applied to the shaft relative to the mounting features (Datum Feature A), hole (Datum Feature B), and slot (Datum Feature C).

$$\text{⌀ } 0.1 \text{ (M) } \text{A} \text{ B} \text{ C}$$

Regardless of the dimension scheme approach, a tolerance, or permissible variation must be determined to ensure that leakage will not occur. After all, it is a given that the shaft will not always be perfectly located with respect to the other features on the motor.

## Integration & CAD Functionality

The key to easily incorporating thermal expansion into a 3D assembly variation model is the unique CAD feature identifier. All features used in a CETOL6 $\sigma$  analysis are tied to a unique geometry IDs in CAD part models. The geometric relationships required for a variation analysis are derived based on the current state of the model, i.e. the location and orientation of the functional features used to construct assembly constraints, measurements, and dimension schemes. Modification of these relationships does not change the geometry IDs. Therefore, a CETOL 6 $\sigma$  analysis updates automatically with part geometry changes. What would you like your readers to understand? Add notes on key takeaways here.

## Incorporating Thermal Expansion

Thermal expansion is the tendency of matter to change in volume in response to a change in temperature. The degree of expansion divided by the change in temperature is referred to as the material's coefficient of thermal expansion.

The **coefficient of thermal expansion** describes how the size of an object changes with a change in temperature. Specifically, it measures the fractional change in size per degree change in temperature at a constant pressure. Common engineering solids usually have coefficients of thermal expansion that do not vary significantly over the range of temperatures where they are designed to be used, so where extremely high accuracy is not required, practical calculations can be based on a constant, average, value of the coefficient of expansion.

The expansion is accounted for by scaling the geometry in CAD model. The scale factor is controlled by the following equation:

$$DS = CTE \cdot (T_1 - T_0)$$

where,

- DS = Change in scale factor ( $S = 1 + DS$ )
- CTE = Coefficient of thermal expansion
- $T_0$  = Reference temperature
- $T_1$  = New temperature

Procedure:

1. Develop a CETOL 6 $\sigma$  model for the assembly you wish to analyze.
2. For each part in the model for which you want to include the effects of thermal expansion, scale the model using available CAD functionality (the method varies based on your CAD system). You can calculate the scale factor using the following equation:

$$S = 1 + \text{CTE} \cdot (T_1 - T_0)$$

3. To analyze at a different temperature, apply a different scale factor to each part as described in step 2. (To get back to the original dimensions, scale the parts by the reciprocal of the previous scale factor.)
4. Update the assembly model using CAD functionality, i.e. regenerate, rebuild, etc.
5. In CETOL 6 $\sigma$ , reconnect with assembly model and recalculate answer(s).

It is worth mentioning that this method has a simplifying assumption to not scale the tolerances by the linear coefficient of expansion. This introduces a small amount of error reducing the accuracy primarily for worst-case results. When multiple dimensions are involved in the loop with 1:1 sensitivity, obviously the loss of accuracy grows and should be considered in very tight applications such as compressors or engines. Since the error is proportional to the magnitude of the tolerance, if the tolerances are relatively small, error is low. The impact to statistical results is mostly insignificant given the error would affect the tails of the distribution for a given dimension.

## Conclusion

Gaining efficiencies in the development of products allows companies to get to the market ahead of the competition. In most product development efforts, time is the most valued resource. For the design engineer, having software tools that are integrated closely with the CAD model results in design reuse and increases the agility of the development team to address all aspects of the product in a shorter period of time. Standardizing on the right product development software tools not only leads to higher quality products and greater market share, but also builds confidence within the company's technical community.